

A Planck-like problem for quantum charged black holes

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Abstract

Motivated by the parallelism existing between the puzzles of classical physics at the beginning of the XXth century and the current paradoxes in the search of a quantum theory of gravity, we give, in analogy with Planck’s black body radiation problem, a solution for the exact Hawking flux of evaporating Reissner-Nordström black holes. Our results show that when back-reaction effects are fully taken into account the standard picture of black hole evaporation is significantly altered, thus implying a possible resolution of the information loss problem.

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The remarkable discovery that black holes emit thermal radiation [1] has raised serious doubts on the unitarity of a quantum theory of gravity. Hawking argued [2] that the semiclassical approximation should be valid until the Planck mass is reached. This, in turn, implies that the black hole should shrink slowly during the evaporation. At the Planck mass there is not enough energy inside the black hole to radiate out the information of the collapsed matter, thus implying a loss of quantum coherence. However, it has been stressed [3] that gravitational back-reaction effects could change the standard picture of the evaporation process. It is clear that the back-reaction must be very important, at least at late times, in order to prevent the total emitted energy to diverge. In contrast, at early times one could expect these effects to be negligible and the radiation can be calculated using the classical space-time geometry.

A natural scenario where one can exactly evaluate the emitted radiation at late times is in the scattering of extremal Reissner-Nordström (RN) black holes by massless neutral particles. If the incoming matter has a long-wavelength, and preserves spherical symmetry, the resulting near-extremal RN black hole can be described, by an infalling observer crossing the horizon, by means of the ingoing Vaidya-type metric [4]

$$ds^2 = -\frac{2l}{r_0} \left(\frac{2x^2}{l^2 q^3} - lm(v) \right) dv^2 + 2dvdx + (r_0^2 + 4lx)d\Omega^2, \quad (1)$$

where $l^2 = G$ is Newton's constant and $r_0 = lq$ is the extremal radius. The function $m(v)$ represents the deviation of the mass from extremality and it verifies the evolution law

$$\partial_v m(v) = -\frac{\hbar}{24\pi lq} m(v) + (\partial_v f)^2, \quad (2)$$

f being the null matter field related to the 2d stress tensor T_{vv}^f , and the corresponding 4d one $T_{vv}^{(4)}$, by $(\partial_v f)^2 \equiv T_{vv}^f = 4\pi r^2 T_{vv}^{(4)}$. Technically, this is the solution coming from the effective action S_{eff} which is obtained by integrating out the field f . Due to spherical symmetry and considering the near-horizon region, S_{eff} corresponds to the classical action plus the Polyakov-Liouville one [5].

In the absence of incoming matter ($m(v) = 0$), and in the very near-horizon limit $r_0^2 \gg lx$, the metric (1) recovers the Robinson-Bertotti anti-de Sitter geometry [6]. In the dynamical situation the metric (1) implies the existence of a negative incoming quantum flux crossing the horizon that goes down exponentially for $v \rightarrow +\infty$. A “complementary” description can also be given from the point of view of an asymptotic observer, for whom there is no incoming radiation but there exists an outgoing (Hawking) evaporation flux. The geometry for the outside observer can be then described, at late times, by an outgoing Vaidya-type metric [7]

$$ds^2 \sim -\frac{2l}{r_0} \left(\frac{2x^2}{l^2 q^3} - lm(u) \right) dv^2 - 2dudx + (r_0^2 + 4lx)d\Omega^2, \quad (3)$$

where $m(u)$ satisfies the equation

$$\partial_u m(u) = -\frac{\hbar}{24\pi lq^3} m(u), \quad (4)$$

which can be integrated easily

$$m(u) = m_0 e^{-\frac{N\hbar}{24\pi lq^3} u}. \quad (5)$$

Requiring that both descriptions match at the end-point $u = +\infty, v = +\infty$ (where the extremal configuration is recovered) we can determine the integration constant m_0

$$m_0 = m(v_f) e^{\frac{N\hbar}{24\pi lq^3} v_f}, \quad (6)$$

where v_f is the value of the advanced time v at which the classical incoming matter is turned off and $m(v_f)$ is calculated using (2) with the condition $m(v = -\infty) = 0$. For finite values of u and v the descriptions of the two observers differ and this, in some sense, is in agreement with the principle of complementarity [3, 8]. Moreover, this expression gives immediately the exact asymptotic behaviour of the Hawking flux at late times $u \rightarrow +\infty$

$$\langle T_{uu}^f(u) \rangle \sim \frac{\hbar}{24\pi lq^3} m(v_f) e^{-\frac{N\hbar}{24\pi lq^3} (u - v_f)}. \quad (7)$$

Note that two different expansions in \hbar are implicit in (7). One is associated to the exponential, fulfilling Stefan's law, and the other inside $m(v_f)$. The second is physically very interesting because it captures details of the incoming matter.

The issue is now to work out the Hawking radiation flux for finite u , but to this end we can no longer use the one-loop effective action that controls the physics near the horizon. Therefore we face directly with the problem that the effective theory describing the whole asymptotic region is unknown. However, the physical intuition suggests the following set of conditions for the Hawking flux $\langle T_{uu}^f \rangle$ (with back-reaction effects included):

1. At early times, $u \rightarrow -\infty$, it must coincide with the Hawking flux calculated ignoring the back-reaction.
2. At late times, $u \rightarrow +\infty$, it should behave as (7).
3. At leading order in \hbar , it must also agree with the Hawking flux computed neglecting the back-reaction.
4. It has to be compatible with energy conservation.

Therefore we deal with a problem similar to that considered by Planck, one century ago, for the black body radiation and the conditions 1 and 2 mimic, respectively, the Rayleigh-Jeans and Wien laws for low and high frequencies of black body emission. They represent small and large back-reaction effects or, in the black body analogy, classical and pure quantum behaviours. Such as in this analogy, what we need now is an interpolating function matching the asymptotic behaviours at early and late times. Note that in our problem we have two additional requirements: at leading order in \hbar the flux should agree with that calculated in the fixed classical background (condition 3) and it has to be compatible with energy conservation (condition 4).

We shall now provide a solution to this problem if the incoming matter is given by a finite set of spherical null shells with energies m_1, \dots, m_N falling into the black hole at the advanced times v_1, \dots, v_N , respectively. Note that in the limit $N \rightarrow \infty$, while keeping $N(v_N - v_1)$ finite, we can imitate a continuous distribution of matter. In this situation the classical stress tensor is given by $T_{vv}^f = \sum_{i=1}^N m_i \delta(v - v_i)$, and it can be written as

$$T_{vv}^f = \sum_{i=1}^N m a_i \delta(v - v_i), \quad (8)$$

where $a_i = \frac{m_i}{m}$, $m = \sum_{i=1}^N m_i$. The classical solution can be determined by matching static solutions at v_1, \dots, v_N . In this way one can write the relation between the initial u_{in} and final u outgoing Eddington-Finkelstein coordinates

$$\begin{aligned} \frac{du}{du_{in}} &= \frac{(r(v_N, u_{N-1}) - r_+^{(N-1)})(r(v_N, u_{N-1}) - r_-^{(N-1)})}{(r(v_N, u) - r_+^{(N)})(r(v_N, u) - r_-^{(N)})} \frac{(r(v_{N-1}, u_{N-2}) - r_+^{(N-2)})}{(r(v_{N-1}, u_{N-1}) - r_+^{(N-1)})} \\ &\quad \frac{(r(v_{N-1}, u_{N-2}) - r_-^{(N-2)})}{(r(v_{N-1}, u_{N-1}) - r_-^{(N-1)})} \dots \frac{(r(v_1, u_{in}) - q)^2}{(r(v_1, u_1) - r_+^{(1)})(r(v_1, u_1) - r_-^{(1)})}, \end{aligned} \quad (9)$$

where $r(v, u_i)$ is the radial function after the i -th null shell and $r_{\pm}^{(i)}$ are the outer and inner horizons of the corresponding static RN black hole.

The Hawking flux without back-reaction $\langle T_{uu}^f \rangle_{NBR}$ is given by the Schwarzian derivative [9]

$$\langle T_{uu}^f(u) \rangle_{NBR} = -\frac{\hbar}{24\pi} \{u_{in}, u\}, \quad (10)$$

and it can be regarded as a function of u , m , v_i and a_i . Let us now consider the differential equation

$$\frac{dm(u)}{du} = -\langle T_{uu}^f \rangle_{NBR}(u, m(u), v_i, a_i), \quad (11)$$

with the initial condition $m(u = -\infty) = \sum m_i = m$, and substitute the constant m for $m(u)$ in the Hawking flux calculated on the fixed classical background $\langle T_{uu}^f \rangle_{NBR}$.

In this way we get a Hawking flux $\langle T_{uu}^f \rangle$ verifying the above four conditions.

For the asymptotic observer the effective metric is given by an outgoing Vaidya metric with mass $m(u)$. Energy conservation requires that

$$\frac{dm(u)}{du} = -\langle T_{uu}^f \rangle, \quad (12)$$

and therefore condition 4 is satisfied by construction. At early times $u \rightarrow -\infty$ and also at leading order in \hbar for all u (no back-reaction), we have $m(u) = m$ and therefore $\langle T_{uu}^f \rangle = \langle T_{uu}^f \rangle_{NBR}$. At late times $u \rightarrow +\infty$ the solution $m(u)$ behaves as

$$m(u) \sim \sum_i m_i e^{-\frac{\hbar}{24\pi q^3}(u-v_i)}, \quad (13)$$

in agreement with condition 2. It is well known that, without back-reaction effects, the Hawking flux $\langle T_{uu}^f \rangle_{NBR}$ approaches the constant thermal value very rapidly, as soon as the exponential late time form of the redshift factor is reached

$$\frac{du}{du_{in}} \sim e^{2\pi T_H u}, \quad (14)$$

where T_H is the Hawking temperature. Once we consider back-reaction effects, due to the interaction between infalling matter and outgoing radiation the relation between the coordinates u_{in} and u is given, in terms of our exact evaporation flux, by the differential equation

$$\langle T_{uu}^f(u) \rangle = -\frac{\hbar}{24\pi} \{u_{in}, u\}. \quad (15)$$

Because $\langle T_{uu}^f(u) \rangle \rightarrow 0$ at $u \rightarrow \pm\infty$, thermality can then be reached only approximately at an intermediate time $u = u_t$ defined by

$$\left. \frac{d}{du} \langle T_{uu}^f(u) \rangle \right|_{u=u_t} = 0. \quad (16)$$

Soon after $u = u_t$, however, we loose the exponential behaviour (14), and for late times $u \rightarrow +\infty$ we have a large deviation

$$\frac{du}{du_{in}} \sim u^2 (A - Be^{-Cu}), \quad (17)$$

where A, B and C are positive integration constants depending on $m(v_f)$.

In conclusion, during a long time period the radiation is non-thermal (before and after u_t) and the amount of emitted energy is big enough to allow the information of the initial state to be released out to future infinity during the evaporation process. Therefore, back-reaction effects are indeed crucial to understand the evaporation of a

black hole. A more detailed study requires numerical computations, opening the way to a systematic analysis to unravel how the details of the incoming matter are encoded in the outgoing radiation. Finally, we want to stress that, following the historical analogy, it would also be interesting to find a theoretical framework capable to reproduce the Hawking flux proposed here for an evaporating RN black hole.

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